Zeno Regime of Collective Emission: Non-Markovianity beyond Retardation

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To build up a collective emission, the atoms in an ensemble must coordinate their behavior by exchanging virtual photons. We study this non-Markovian process in a subwavelength atom chain coupled to a one-dimensional (1D) waveguide and find that retardation is not the only cause of non-Markovianity. The other factor is the memory of the photonic environment, for which a single excited atom needs a finite time, the Zeno regime, to transition from quadratic decay to exponential decay. In the waveguide setup, this crossover has a time scale longer than the retardation, thus impacting the development of collective behavior. By comparing a full quantum treatment with an approach incorporating only the retardation effect, we find that the field memory effect, characterized by the population of atomic excitation, is much more pronounced in collective emissions than that in the decay of a single atom. Our results maybe useful for the dissipation engineering of quantum information processings based on compact atom arrays.

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It is well known since the 1960s [1,2] that the short-time dynamics of an excited atom differs significantly from the exponential decay based on the Weisskopf-Wigner formalism [3]. The finite memory of the photonic reservoir leads to a growth in the decay rate from zero that is quadratic in time [4,5]. It has inspired attempts to prevent decay by quickly repeating measurements [6–9], i.e., the Zeno effect. Actually, the duration of the nonexponential decay, characterized by the Zeno time τ_Z, is typically many orders of magnitude shorter than the lifetime, rendering the field memory effect undetectable. For example, an optimal estimation of the $2P$-$1S$ transition of the hydrogen atom reads $\tau_Z \sim 10^{-13}$ s [10]. Instead of the decay of a single atom, in this Letter, we study atom ensembles [11], especially the subwavelength atom arrays, where the separation between two adjacent atoms, d, is shorter than the resonant wavelength λ, and reveal the prominent memory effect in the Zeno regime.

The long-time collective emissions from an atomic ensemble are well described by Lehmberg's formalism based on the Markov approximation [12,13]. For example, it predicts a power-law scaling $\gamma \propto N^{-\alpha}$, with N the number of atoms for the subradiant states of a subwavelength atom array [14–17]. However, Markovian theories are not able to answer how the collective behaviors are built up. This process must be non-Markovian because the atoms are organized by the retarded photon-mediated interactions. Instead, we may upgrade the Markovian description minimally by including delayed feedback: Every atom starts from exponential decay independently but adjusts its decay rate in response to the signal from another atom. This physical picture has been analytically studied for two atoms with the photonic reservoir being 3D free space [18] or 1D waveguide [19–21]. Retardation effects of waveguide quantum electrodynamics (QED) are also studied in Refs. [22–27].

But how does the Zeno regime come into effect? The Zeno time, exemplified by the $2P$-$1S$ transition of the hydrogen atom satisfies $\tau_Z \gg 2\pi/\omega_0 \sim 10^{-15}$ s [10]. In a subwavelength atom array, the minimal retardation time $t_{\mathrm{retard}} = d/c$, with c the speed of light, fulfills that $t_{\mathrm{retard}} < \lambda/c = 2\pi/\omega_0 \ll \tau_Z$. It means that the virtual photons sent from an atom have already passed many other atoms, building up cooperativeness to a certain extent, while an isolated atom has not yet entered exponential decay. Thus, the development of the full collective emission and the reduction to Markovian behavior are two simultaneous processes highly intertwined. As illustrated in Fig. 1(a), the above-mentioned retardation-only picture studied in Refs. [19–27] does not apply to compact atom ensembles, i.e., there are memory effects beyond retardation.

To show the memory effect beyond retardation, we shall compare the retardation-only picture with a full quantum treatment. Their difference is displayed by the evolution of instantaneous decay rates and excited state populations. Our findings indicate that this memory effect in collective emissions is much more pronounced than that in the decay of a single atom, making subwavelength atom array a better platform to detect the non-Markovianity in the Zeno regime. In this Letter, we shall concentrate on the setup of waveguide QED. Experimental feasibilities and memory effect of free space radiation field will also be discussed.

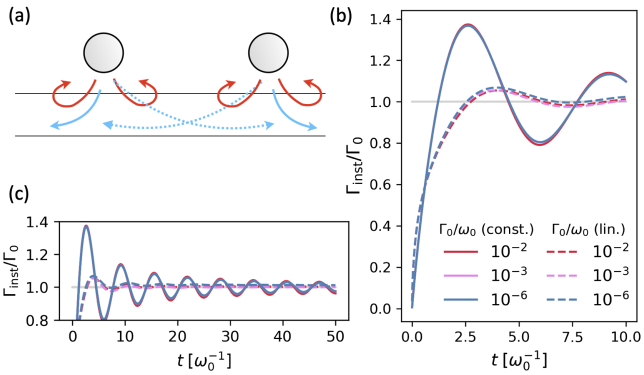

FIG. 1. (a) Sketch of the idea: For closely separated atoms, development of collective behaviors by exchanging virtual photons (dotted blue arrows) and the transition from quadratic to exponential decay of a single atom (Zeno regime caused by the memory effect of the field, represented by the red backflow arrows) are highly intertwined. (b) The instantaneous emission rate $\Gamma_{\rm inst}(t)$ of the decay of a single atom (in units of Γ_0). Legend: const-wQED (solid curves), lin-wQED (dashed curves). Values of Γ_0/ω_0 are distinguished by coloring: 10^{-2} (red), 10^{-3} (pink), and 10^{-6} (blue). (c) Enlarged view of (b) for $t \leq 50/\omega_0$.

Zeno time of the waveguide QED.—The waveguide QED setup consists of N two-level atoms with the ground state $|g\rangle$ and the excited state $|e\rangle$, and a 1D continuum of bosonic modes. The annihilation and generation operator of the waveguide mode with wave number k are denoted by a_k and $a_k^\dagger$, respectively. They satisfy the bosonic commutation relation $[a_k, a_{k'}^\dagger] = 2\pi\delta(k-k')$. The Hamiltonian of the system is conventionally written in analogy with the multipolar gauge Hamiltonian of quantum optics, colloquially the "$\boldsymbol{d}\cdot\boldsymbol{E}$" Hamiltonian [28–30], though the bosons field may not be photonic, e.g., it could be surface acoustic waves [31,32] and matter waves [33,34], etc.,

$$H_M = \sum_{i=1}^{N}\omega_0\sigma_i^\dagger\sigma_i + \int_{-\Lambda}^{\Lambda}\frac{dk}{2\pi}\omega_k a_k^\dagger a_k - i\sum_{i=1}^{N}\int_{-\Lambda}^{\Lambda}\frac{dk}{2\pi}g_k\sigma_{i,X}(a_k e^{ikx_i} - a_k^\dagger e^{-ikx_i}), \quad (1)$$

where $\sigma_i = |g\rangle_i\langle e|$, $\sigma_{i,X} = \sigma_i + \sigma_i^\dagger$, x_i denotes the coordinate of atom i, Λ is the cutoff of the wave number. The coupling strength g_k will be specified below. Here we assume a linear dispersion relation for the waveguide, $\omega_k = v_g|k|$, where v_g is the group velocity of the guided modes. (The Zeno time of a setup with nonlinear dispersion relation is found qualitatively the same [35].) The atomic transition frequency ω_0 defines a resonant wave number $k_0 = \omega_0/v_g$. We assume $k_0 \ll \Lambda$ so that the non-Markovianity induced by reservoir band edges [38] is irrelevant.

The counterrotating terms of Hamiltonian (1) cannot be ignored for short-time dynamics [10]. Moreover, they are found to lead to nonlinearity at the single-photon level [39]. Fortunately, theoretical difficulties brought by them can be avoided by turning to the gauge introduced by Drummond [40], which is also called the Jaynes-Cummings (JC) gauge [36,41]. The transformation from the multipolar gauge to the JC gauge reads $H_{\rm JC} = e^{-iS}H_M e^{iS}$, where

$$S = \sum_{i=1}^{N}\int_{-\Lambda}^{\Lambda}\frac{dk}{2\pi}\frac{g_k}{\omega_0+\omega_k}\sigma_{i,X}(a_k e^{ikx_i} + a_k^\dagger e^{-ikx_i}). \quad (2)$$

At the first order of g_k, we obtain

$$H_{\rm JC} \approx \sum_{i=1}^{N}\omega_0\sigma_i^\dagger\sigma_i + \int_{-\Lambda}^{\Lambda}\frac{dk}{2\pi}\omega_k a_k^\dagger a_k - i\sum_{i=1}^{N}\int_{-\Lambda}^{\Lambda}\frac{dk}{2\pi}g_k^{\rm JC}(\sigma_i^\dagger a_k e^{ikx_i} - \sigma_i a_k^\dagger e^{-ikx_i}), \quad (3)$$

where $g_k^{\rm JC} = 2g_k\omega_0/(\omega_k+\omega_0)$. The neglected terms at the order of $O(g_k^2)$ give corrections to the atom and waveguide self-energies, while corrections to their couplings occur at the order of $O(g_k^3)$ [10,42,43].

The absence of counterrotating terms grants $H_{\rm JC}$ (3) a nice property that its ground state is identical to that of its free part $|G\rangle = |g_1, g_2 \cdots g_N, \emptyset\rangle$, an atom-field product state, where $\emptyset$ denotes the field vacuum. Then, the physical state of exciting an atom from the overall ground state, $\sigma_i^\dagger|G\rangle$, is also a product state. This kind of physical states are exactly the initial states that we are interested in. With respect to the multipolar gauge H_M, the same physical state is written as $e^{iS}\sigma_i^\dagger|G\rangle$, which is, however, an entangled state between the atoms and the field. Recall that initial states in the product form can greatly simplify the theoretical analysis and are essential for theories of open quantum system [44–46]. Thus, we choose to work with $H_{\rm JC}$ instead of H_M.

Next, let us specify the coupling strength g_k. In the literature of waveguide QED, a localized atom-field interaction is often assumed [47] so that g_k is a k-independent constant [28–30,47]. It can be spelled by the Markovian decay rate Γ_0 as $g_k^2 = \Gamma_0 v_g/2$. Another option for g_k is $g_k \propto |k|^{1/2}$, the same as the multipolar Hamiltonian of atoms in free space [30] (recall that an atom does not couple to the full displacement field but only its transverse component, which is a nonlocal field [48]). In this case, we have $g_k^2 = \Gamma_0 v_g|k|/(2k_0)$. The above two choices for g_k correspond to a constant and a linear spectral density, hence will be denoted by "const-wQED" and "lin-wQED," respectively.

We are now in a position to calculate the Zeno time. Given an initial state $|\Psi_0\rangle$ and a Hamiltonian H, the Zeno time τ_Z is defined from the short-time expansion of the nondecay probability

$$\left|\langle\Psi_0|e^{-iHt}|\Psi_0\rangle\right|^2 = 1 - t^2/\tau_Z^2 + \cdots. \quad (4)$$

The Zeno time is a characterization to the duration of nonexponential decay, but not an exact measure. Nevertheless, we substitute $|\Psi_0\rangle = |e, \emptyset\rangle$ and $H_{\rm JC}$ with $N = 1$ into Eq. (4) and obtain

$$\tau_Z^{-2} = \begin{cases} 2\Gamma_0\omega_0/\pi & \text{const} \\ 2\Gamma_0\omega_0\ln(\Lambda/k_0)/\pi & \text{lin} \end{cases} \quad (5)$$

Remarkably, τ_Z of const-wQED is independent of the cutoff Λ, which is introduced in Eq. (1). This is not seen elsewhere. The above result implies that $\tau_Z \gg 1/\omega_0$, hence $\tau_Z \gg \tau_{\rm retard}$, is valid if $\omega_0/\Gamma_0 \gg 1$ (for const-wQED) or $\omega_0/\Gamma_0 \gg \ln(\Lambda/k_0)$ (for lin-wQED). Such weak atom-field couplings are satisfied commonly. Zeno time for $N > 1$ is discussed in Ref. [35].

Equation of motion.—Suppose that the system is initialized with only one atomic excitation. Note that $H_{\rm JC}$ preserves the number of excitations, thanks to the absence of counterrotating terms. Thus, the evolution is captured by the singly excited ansatz

$$|\Psi(t)\rangle = \sum_{i=1}^{N}\alpha_i(t)\sigma_i^\dagger|G\rangle + \int_{-\Lambda}^{\Lambda}\frac{dk}{2\pi}\beta_k(t)a_k^\dagger|G\rangle, \quad (6)$$

where $\alpha_i(t)$ and $\beta_k(t)$ are superposition coefficients to be determined. The Schrödinger equation in the interaction picture implies the integro-differential equation

$$\frac{d}{dt}\alpha_i(t) = -\sum_{j=1}^{N}\int_{-\Lambda}^{\Lambda}\frac{dk}{2\pi}\left|g_k^{\rm JC}\right|^2\int_0^t d\tau\,\alpha_j(\tau) \times e^{ik(x_i-x_j)+i(\omega_k-\omega_0)(t-\tau)}. \quad (7)$$

This equation is further transformed into an integral equation and solved numerically [35].

The above equation will be compared with the following one embodying only the non-Markovianity caused by retardation [20,21]

$$\frac{d\alpha_i(t)}{dt} = -\frac{\Gamma_0}{2}\left[\alpha_i(t) + \sum_{j\neq i}e^{ik_0r_{ij}}\alpha_j\left(t-\frac{r_{ij}}{v_g}\right)\Theta\left(t-\frac{r_{ij}}{v_g}\right)\right], \quad (8)$$

where $r_{ij} = |x_i - x_j|$ and $\Theta(t) = 1$ for $t > 0$ and vanishes otherwise. It can be derived from Eq. (7) via an approximation introduced in Ref. [18], see also Ref. [35]. Note that while the right-hand side of Eq. (7) incorporates the entire history $\tau \in [0, t]$, the right-hand side of Eq. (8) includes only a distance-dependent delay. Ignoring this delay immediately aligns it with the Markovian effective non-Hermitian Hamiltonian of waveguide QED [30]. Hereafter, data produced by Eq. (8) will be labeled by "retard."

We shall characterize the nonexponential decay by instantaneous decay rate $\Gamma_{\rm inst}$ and population in excited state $P_e(t)$ of the whole chain:

$$\Gamma_{\rm inst}(t) \equiv -\frac{d}{dt}\ln P_e(t), \qquad P_e(t) = \sum_{i=1}^{N}|\alpha_i(t)|^2. \quad (9)$$

These two quantities can also be defined for every individual atom in an apparent way.

Individual decay.—Let us start from the decay of a single atom. We plot $\Gamma_{\rm inst}(t)$ in units of Γ_0 in Figs. 1(b),1(c) for both const-wQED (solid curves) and lin-wQED (dashed curves). Either one is calculated with three values of Γ_0/ω_0, 10^{-2} (red), 10^{-3} (pink), and 10^{-6} (blue). The cutoff is set at $\Lambda/k_0 = 10^4$. For either const-wQED or lin-wQED, curves belonging to the three Γ_0/ω_0 almost overlap; for each value of Γ_0/ω_0, $\Gamma_{\rm inst}(t)$ of lin-wQED increases faster at first ($t \lesssim 1/\omega_0$) but soon becomes more gradual than that of const-wQED. The latter increases to roughly $1.4\Gamma_0$ and turns to oscillating around Γ_0 with a waning amplitude. The non-Markovianity of const-wQED is more pronounced, as what we learn from its Zeno time (5). It is shown in Fig. 1(c) that the oscillation of the curves of const-wQED is still visible at $t = 50/\omega_0$, equivalent to a distance of eight wavelengths for photon propagation.

Although the curves of $\Gamma_{\rm inst}(t)$ clearly demonstrate the nonexponential decay, it is defined as the derivative with respect to time so that producing the curves requires a high temporal resolution ($\ll 1/\omega_0$) of measuring $P_e(t)$. This is of course experimentally challenging. The requirement of temporal resolution might be relaxed if the non-Markovianity can be manifested by $P_e(t)$ itself, or equivalently, $\Delta P_e(t) = P_e(t) - P_e(0)$. Unfortunately, we will see in Fig. 2(e) that it is not the case for $N = 1$: $\Delta P_e(t)$ is averaged out to the memory-less Markovian result quickly. But fortunately, it would be possible for subwavelength atom arrays ($N > 1$).

Superradiance.—We consider a chain of atoms initialized in the timed-Dicke state

$$|\Psi_k\rangle = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}e^{ikx_j}\sigma_j^\dagger|G\rangle. \quad (10)$$

This kind of states are experimentally accessible [49,50]. State (10) with $k = \pm k_0$ are the single-photon superradiant state [51]. We substitute $|\Psi_{k_0}\rangle$ with $N = 20$, $\Gamma_0/\omega_0 = 10^{-4}$, and atom-atom separation $d = 0.1\pi/k_0$ (see results of $d = 0.5\pi/k_0$ in [35]) into Eqs. (7) and (8) and show the

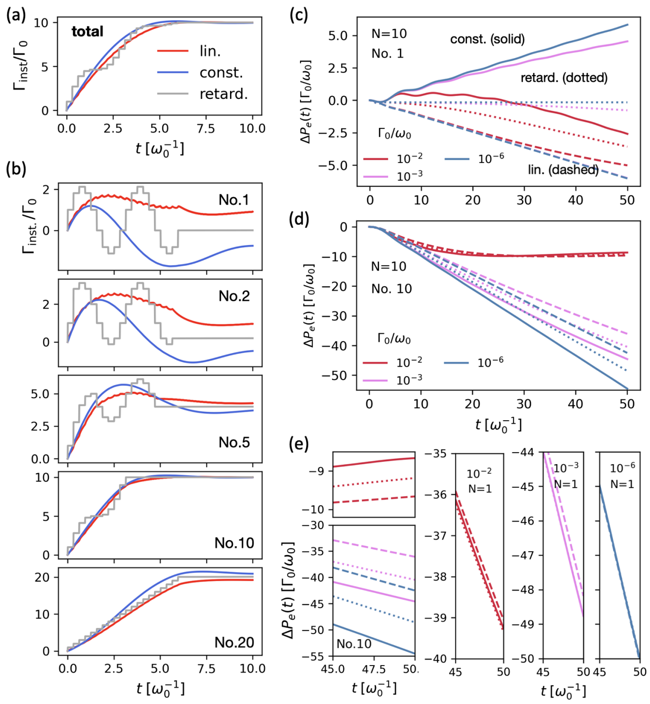

FIG. 2. The decay of superradiant state $|\Psi_{k_0}\rangle$. (a) The instantaneous decay rate $\Gamma_{\rm inst}(t)$ for const-wQED (red) and lin-wQED (blue), both of which are determined by Eq. (7), and the retardation-only solution retard (gray) given by Eq. (8). Other parameters: $\Gamma_0/\omega_0 = 10^{-4}$, $N = 20$, and $d = 0.1\pi/k_0$. (b) The individual instantaneous decay rate of five selected atoms [the legend is the same as in (a)]. The change of individual excited state population $\Delta P_e(t)$ (in units of Γ_0/ω_0) for (c) atom 1; (d) atom 10. In (c),(d), we have $N = 10$, $d = 0.1\pi/k_0$ and $\Gamma_0/\omega_0 = 10^{-2}$ (colored by red) 10^{-3} (pink), and 10^{-6} (blue). Predictions of const-wQED, lin-wQED and the retardation-only solution retard are plotted by solid, dashed, and dotted curves, respectively. (e) Left panel: Enlargement of (d) for $\omega_0 t \in [45, 50]$; the three right panels: $\Delta P_e(t)$ of a single atom coupled to the waveguide with $\Gamma_0/\omega_0 = 10^{-2}$, 10^{-3} and 10^{-6}, respectively.

results of $\Gamma_{\rm inst}(t)$ in Fig. 2(a) for $t \le 10/\omega_0$. Curves of Eq. (7) (red for lin-wQED and blue for const-wQED) show continuous growth while that of Eq. (8) (gray) gives a steplike increase. They agree well after $t \approx 6/\omega_0$.

Next, we pick five atoms, No. 1, 2, 5, 10, 20 ($x_i < x_j$ if $i < j$), and plot the individual instantaneous decay rate, $\Gamma_{j,{\rm inst}}(t) = -d(\ln|\alpha_j|^2)/dt$ in Fig. 2(b). It shows that the atoms decay at different rates. Atom 1 decays slowly while atom 20 accelerates to $20\Gamma_0$ (the opposite is obtained if we choose $|\Psi_k\rangle$ with $k = -k_0$). The three curves (const-wQED, lin-wQED and retard) agree better for atoms in the middle, i.e., atom 10. In particular, for atom 1, significant derivations between three curves of $\Gamma_{1,{\rm const}}$ are visible: The gray curve (retard) shows two cycles of emission and absorption, the red curve (const-wQED) shows only emission while absorption is dominant for the blue curve (lin-wQED).

Such discrepancy inspires us to look at the change of individual population $\Delta P_e = |\alpha_j(t)|^2 - |\alpha_j(0)|^2$, where $|\alpha_j(0)|^2 = 1/N$ for state (10). In Fig. 2(c), we plot it for atom 1 of a shorter chain ($N = 10$) within a longer time window $t \le 50/\omega_0$. To compare with Fig. 1, we apply the same three values of Γ_0/ω_0 and the same coloring as in Fig. 1. Figure 2(c) shows that for $\Gamma_0/\omega_0 = 10^{-3}$ (pink) and 10^{-6} (blue) there is a gap of $\lesssim 10\Gamma_0/\omega_0$ between const-wQED (solid curves) and lin-wQED (dashed curves), while the predictions of Eq. (8) (dotted curves) are roughly in the middle. For stronger atom-waveguide coupling $\Gamma_0/\omega_0 = 10^{-2}$, the curves of const-wQED (red solid) and that of Eq. (8) (red dotted) have a tendency toward getting closer. We also plot $\Delta P_e(t)$ for the last atom (No. 10) in Fig. 2(d). It shows gaps between const-wQED (solid) and lin-wQED (dashed) of roughly the same scale as atom 1, except for the case of $\Gamma_0/\omega_0 = 10^{-2}$. An enlarged view for $\omega_0 t \in [45, 50]$ is shown in the left-most panel of Fig. 2(e).

To compare, we plot $\Delta P_e(t)$ for the case of $N = 1$ in the right three panels of Fig. 2(e), each for one choice of Γ_0/ω_0. We find that predictions of the three models (solid, dashed, and dotted curves) are almost indistinguishable for $\omega_0 t \in [45, 50]$. Thus, in terms of $\Delta P_e(t)$, the non-Markovian effect in the Zeno regime is much more prominent in collective emissions than in the decay of a single atom. And it is reasonable to conclude that the requirement of temporal resolution is significantly relaxed to the level of $\lesssim 10/\omega_0$.

Subradiance.—The effective Hamiltonian of waveguide QED defines a subradiant eigenstate approximated by $|\Psi_{\rm sub}\rangle = (|\Psi_k\rangle - |\Psi_{-k}\rangle)/\sqrt{2}$ with $kd = \pi N/(N+1)$ [14–16]. We suppose the same parameters as in Figs. 2(a) and 2(b) and plot $\Gamma_{\rm inst}(t)$ for a chain initialized in $|\Psi_{\rm sub}\rangle$ in Fig. 3(a). In all cases, the subradiance is built through quick oscillations between emissions and absorptions. But the amplitudes are different: The piecewise curve

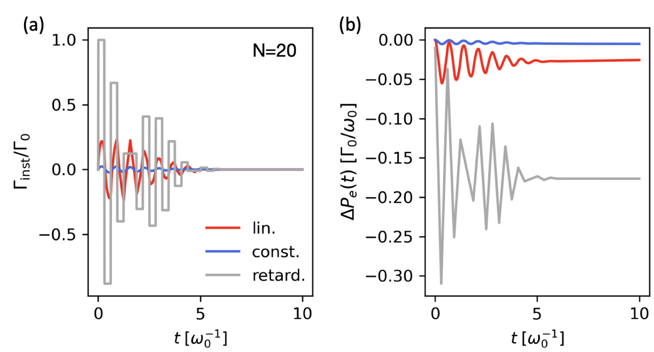

FIG. 3. The decay of the subradiant state $|\Psi_{\rm sub}\rangle$ with $N = 20$. (a) The instantaneous decay rate $\Gamma_{\rm inst}(t)$ (in units of Γ_0). The other parameters and the legend are the same as in Fig. 2(a). (b) The change of population on the excited state $\Delta P_e(t)$ (in units of Γ_0/ω_0).

predicted by Eq. (8) has the largest amplitudes. In Fig. 3(b), we plot $\Delta P_e(t)$ of the whole chain. It also shows apparent relative discrepancies between the three predictions.

Discussions.—It is of fundamental interest to extend the theory to atoms in free space. However, a controversial issue is that on which Hamiltonian all calculations should be based. Most works chose the Coulomb gauge ($\boldsymbol{A}\cdot\boldsymbol{p}$ interaction) disregarding the $\boldsymbol{A}^2$ term, see, e.g., Refs. [52–56]. Taking the counterrotating terms into account, it has been found that $\tau_Z^{-2} \sim \ln\Lambda$ [10], the same as lin-wQED. Thus, we expect the same non-Markovianity, or perhaps even more pronounced, because the resonant dipole-dipole interaction in free space diverges as $1/r^3$ for short distances, resulting in strong photon blockade [57]. But recently Hamiltonians of quantum optics is revisited by causal perturbation theory [58]. In this sense, non-Markovianity beyond retardation might be viewed as a probe to determine which theory better captures the true physics.

For experimental tests, our plots show that temporal resolution at the scale of $\lesssim 10/\omega_0$ is favorable. Among various platforms of waveguide QED, superconducting circuits have the highest coupling efficiency [30] and fabricating the transmon qubits into a subwavelength chain is straightforward [59]. The transition frequency ω_0 is in the GHz regime so that temporal resolution at nanosecond is sufficient. Subwavelength atom arrays can also be realized by trapping Sr atoms in optical lattices [60]. The wavelength of $^3P_0 - {}^3D_1$ transition is 2.6 μm so the temporal resolution should be $\lesssim 10$ fs. Scenarios where the boson fields are surface acoustic waves [31,32] and matter waves [33,34] need further studies.

Conclusions.—We have studied the Zeno regime of the decay of subwavelength atom arrays coupled to a 1D waveguide. Non-Markovianity beyond retardation, characterized by instantaneous decay rates and population in the excited states, is addressed by comparing the full quantum treatment Eq. (7) with Eq. (8), which includes only the retardation effect. Specifically, the evolution of excited state population (in the single-photon superradiant state) manifests reservoir memory effect with a significantly relaxed temporal resolution. Our results might be useful for protecting the quantum information stored in compact atom ensembles [14,61] via dissipation engineering [62], and studying the correlated noise in quantum computing processors [63], etc. For future works, one may explore such possibilities using the theoretical tools of non-Markovian open systems [44–46].

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